Quantitative Assessment of Bias Sensitivity of Performance Measures for Dichotomous Forecasts

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Reference

This presentation is based mostly on a paper accepted for publication in ***Weather and Forecasting***, now available on line at the American Meteorological Society Publications page under the link "**Early Online Releases** of Papers in Press (2008)." Select the link for *Weather and Forecasting,* and look for the **posting date of July 23, 2008.**

Overview

- Review definitions and 2X2 contingency table
- State motivations and goals
- Derive a Critical Performance Ratio (CPR) that quantifies bias sensitivity
- Apply the CPR to reveal bias dependencies of several performance metrics
- Summarize and discuss future work

Definitions

- **Dichotomous forecasts:** "yes" or "no" forecasts for occurrence of some event, e.g., precipitation accumulation exceeding a threshold
- **Bias (*B*, frequency bias):** the ratio of number or frequency of "yes" forecasts to "yes" observations
- **Probability of Detection (*P*, POD):** the ratio of number or frequency of *correct* "yes" forecasts to "yes" observations
- **Event frequency** (α)**:** the fraction of the entire verification domain (temporal and spatial) comprised of "yes" observations

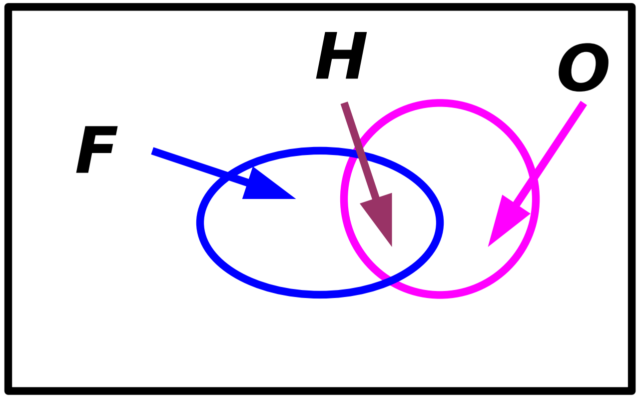
Contingency Table

EVENTS	Observed	Not Observed	Total
Forecast	H	$F-H$	F
Not Forecast	$O-H$	$N-F-O+H$	$N-F$
Total	O	$N-O$	N

Where:

H = number or area of correct "yes" forecasts, hits
F = number or area of "yes" forecasts
O = number or area of "yes" observations
N = total number or area constituting the verification domain

$$B = \frac{F}{O} \qquad P = \frac{H}{O} \qquad \alpha = \frac{O}{N} > 0$$



Motivations and Goals

- **MOTIVATIONS**: Performance measures computed from *F, H, O* values are known from experience to be sensitive to bias (*e.g*., Baldwin and Kain, 2006), having implications for
  - Assessing "hedged" forecasts
  - Assessing bias correct forecasts
  - Assessing forecasts evaluated using Spatial Techniques
- **GOALS**:
  - Derive a general mathematical expression quantifying bias sensitivity
  - Apply the general quantitative expression to specific performance measures

Analytical Approach

- Rewrite contingency table in terms of *P, B,* and α
- Express a performance measure in terms of *P, B,* and α as independent variables
- Assume verification (retrospective) point of view so that α may be considered constant
- Use total derivative of a performance measure to determine how *P* must change with *B* for the performance measure to indicate improvement

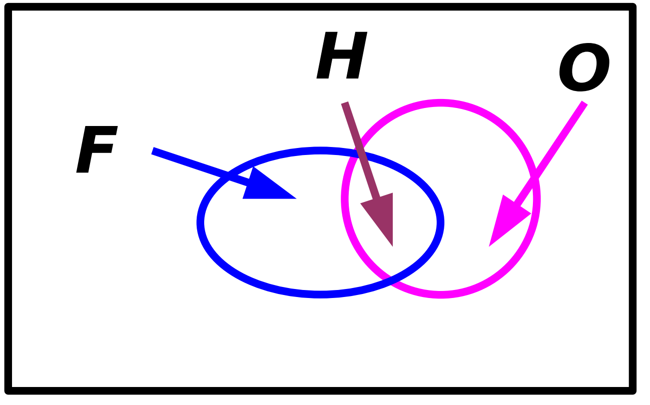
Rewrite Contingency Table

EVENTS	Observed	Not Observed	Total
Forecast	H	$F-H$	F
Not Forecast	$O-H$	$N-F-O+H$	$N-F$
Total	O	$N-O$	N

Multiply each cell by *O/O=1.* Normalize by dividing by *N.*
Replace *H/O* with *P, F/O* with *B,* and *O/N* with α.

EVENTS	Observed	Not Observed	Total
Forecast	αP	$\alpha(B-P)$	αB
Not Forecast	$\alpha(1-P)$	$1-\alpha(B+1-P)$	$1-\alpha B$
Total	α	$1-\alpha$	1

$$B = \frac{F}{O} \qquad P = \frac{H}{O} \qquad \alpha = \frac{O}{N} > 0$$



Mathematical Analysis

Write any performance measure as $S = S(B, P)$, with α constant.

Express total differential of *S:* $dS = (\frac{\partial S}{\partial B})_P \, dB + (\frac{\partial S}{\partial P})_B \, dP$. (1)

Consider ***positively oriented*** performance measures that indicate improvement by increasing in value. Thus, for small changes in *B* and *P* , *S* indicates improvement if

$$(\frac{\partial S}{\partial B}) \, \Delta B + (\frac{\partial S}{\partial P}) \, \Delta P > 0. \qquad (2)$$

It follows that

$$(\frac{\partial S}{\partial P}) \, \Delta P > -(\frac{\partial S}{\partial B}) \, \Delta B. \qquad (3)$$

Since $H = \alpha P$ and $F = \alpha B$, if α is constant, then $\Delta P = \Delta H / \alpha$

and $\Delta B = \Delta F / \alpha$.

Mathematical Analysis Continued

$$\left(\frac{\partial S}{\partial P}\right)\frac{\Delta H}{\alpha} > -\left(\frac{\partial S}{\partial B}\right)\frac{\Delta F}{\alpha} \;. \qquad (4)$$

Consider an increase in bias, $\Delta F > 0$, assuming $(\partial S / \partial P) > 0$,

then the general condition for *S* increase is

$$\frac{\Delta H}{\Delta F} > -\frac{(\frac{\partial S}{\partial B})}{(\frac{\partial S}{\partial P})} = \rho \;. \qquad (5)$$

For a decrease in bias, $\Delta F < 0$, still assuming $(\partial S / \partial P) > 0$,

the general condition for *S* increase is

$$\frac{\Delta H}{\Delta F} < \rho \;. \qquad (6)$$

Mathematical Analysis Continued

$$\rho = -\frac{\left(\frac{\partial S}{\partial B}\right)}{\left(\frac{\partial S}{\partial P}\right)}$$ defines the critical performance ratio (CPR). (6)

In summary, if bias is increased, a performance measure indicates improvement if

$$\text{hit fraction for added forecasts} = \frac{\Delta H}{\Delta F} > \rho . \qquad (7)$$

For a decrease in bias, a performance measure indicates improvement if

$$\text{hit fraction of removed forecasts} = \frac{\Delta H}{\Delta F} < \rho . \qquad (8)$$

The same conditions for *S* to improve obtain for negatively oriented performance measures.

Ineq. (7) or (8) expresses the CPR criterion for a performance measure to improve for a change in bias.

For a decrease in bias, the CPR sets a bar to get under.

For an increase in bias, the CPR sets a bar to get over.

Mathematical Analysis Concluded

The following three constraints apply:

1. $0 \leqslant \dfrac{\Delta H}{\Delta F} \leqslant 1$

   Additional (removed) hits cannot exceed the change in forecast number or area for an increase (decrease) in bias.

2. $\dfrac{\Delta H}{\Delta F} = 0$ If *P=1* and $\Delta B > 0$

3. $\dfrac{\Delta H}{\Delta F} = 0$ If *P=0* and $\Delta B < 0$

If the CPR condition requires a violation of one of these constraints, the performance measure cannot indicate improvement.

Analysis Method Summary

1. Express *S* in terms of *B*, *P*, and $\alpha > 0$.
2. Derive and simplify required partial derivatives.
3. Evaluate $(\partial S / \partial P)$ to assure correct algebraic sign.
4. Compute the CPR, ρ.
5. Select the appropriate inequality based on the sign of the bias change.
6. If the CPR criterion violates any one of the three constraints, the performance measure cannot indicate improvement.

CPRs may be derived and examined graphically as functions of *B*, *P*, and/or α.

Table of Derivative & CPR Formulas for Selected Performance Measures

PM	$S(B,P)$	$\partial S / \partial B$	$\partial S / \partial P$	**CPR**	**CPR for** $B=1$
TS	$\frac{P}{(B+1-P)}$	$\frac{-P}{(B+1-P)^2}$	$\frac{B+1}{(B+1-P)^2}$	$\frac{P}{(B+1)}$	$\frac{P}{2}$
ETS	$\frac{P-\alpha B}{(B+1-P-\alpha B)}$	$\frac{P(2\alpha-1)-\alpha}{(B+1-P-\alpha B)^2}$	$\frac{B+1-2\alpha B}{(B+1-P-\alpha B)^2}$	$\frac{P+\alpha-2\alpha P}{(B+1-2\alpha B)}$	$\frac{P+\alpha-2\alpha P}{2(1-\alpha)}$
HSS	$\frac{2(P-\alpha B)}{(B+1-2\alpha B)}$	$\frac{2P(2\alpha-1)-2\alpha}{(B+1-2\alpha B)^2}$	$\frac{2}{B+1-2\alpha B}$	$\frac{P+\alpha-2\alpha P}{(B+1-2\alpha B)}$	$\frac{P+\alpha-2\alpha P}{2(1-\alpha)}$
CSS	$\frac{P-\alpha B}{B(1-\alpha B)}$	$\frac{2\alpha PB-P-\alpha^2 B^2}{B^2(1-\alpha B)^2}$	$\frac{1}{B(1-\alpha B)}$	$\frac{P+\alpha^2 B^2-2\alpha PB}{B(1-\alpha B)}$	$\frac{P+\alpha^2-2\alpha P}{(1-\alpha)}$
ORSS	$\frac{P-\alpha B}{D}$, $D=P-2\alpha PB-2\alpha P+2\alpha P^2+\alpha B$	$\frac{2\alpha P(P-1)(1-\alpha)}{D^2}$	$\frac{2\alpha Y}{D^2}$, $Y=B-P^2-\alpha B^2-\alpha B+2\alpha BP$	$\frac{P(1-P)(1-\alpha)}{Y}$	$\frac{P(1-\alpha)}{(1+P-2\alpha)}$
TSA dHdF	$\frac{1-(1-P)^{1/B}}{1+(1-P)^{1/B}}$	$\frac{2(1-P)^{1/B}\ln(1-P)}{B^2[1+(1-P)^{1/B}]^2}$	$\frac{2(1-P)^{\frac{1}{B}-1}}{B[1+(1-P)^{1/B}]^2}$	$\frac{(P-1)\ln(1-P)}{B}$	$(P-1)\ln(1-P)$
ETSA dHdF	$\frac{1-\alpha-(1-P)^{1/B}}{1-\alpha+(1-P)^{1/B}}$	$\frac{2(1-\alpha)(1-P)^{1/B}\ln(1-P)}{B^2[1+(1-P)^{1/B}]^2}$	$\frac{2(1-\alpha)(1-P)^{\frac{1}{B}-1}}{B[1+(1-P)^{1/B}]^2}$	$\frac{(P-1)\ln(1-P)}{B}$	$(P-1)\ln(1-P)$
TSA dHdA	$\frac{1-u^{-1}W(u)}{1+u^{-1}W(u)}$, $u=\frac{-\ln(1-P)}{(B-P)}$	$\frac{2[W(u)]^2[1+W(u)]^{-1}}{\ln(1-P)[1+u^{-1}W(u)]^2}$	$-\frac{\partial S}{\partial B}\left(\frac{(1-P)u+1}{(1-P)u}\right)$	$\frac{(P-1)\ln(1-P)}{B-P+(P-1)\ln(1-P)}$	$\frac{\ln(1-P)}{\ln(1-P)-1}$
ETSA dHdA	$\frac{(1-\alpha)-u^{-1}W(u)}{(1-\alpha)+u^{-1}W(u)}$, $u=\frac{-\ln(1-P)}{(B-P)}$	$\frac{2(1-\alpha)[W(u)]^2[1+W(u)]^{-1}}{\ln(1-P)[1+u^{-1}W(u)]^2}$	$-\frac{\partial S}{\partial B}\left(\frac{(1-P)u+1}{(1-P)u}\right)$	$\frac{(P-1)\ln(1-P)}{B-P+(P-1)\ln(1-P)}$	$\frac{\ln(1-P)}{\ln(1-P)-1}$

Mesinger (2008) bias adjusted TS & ETS

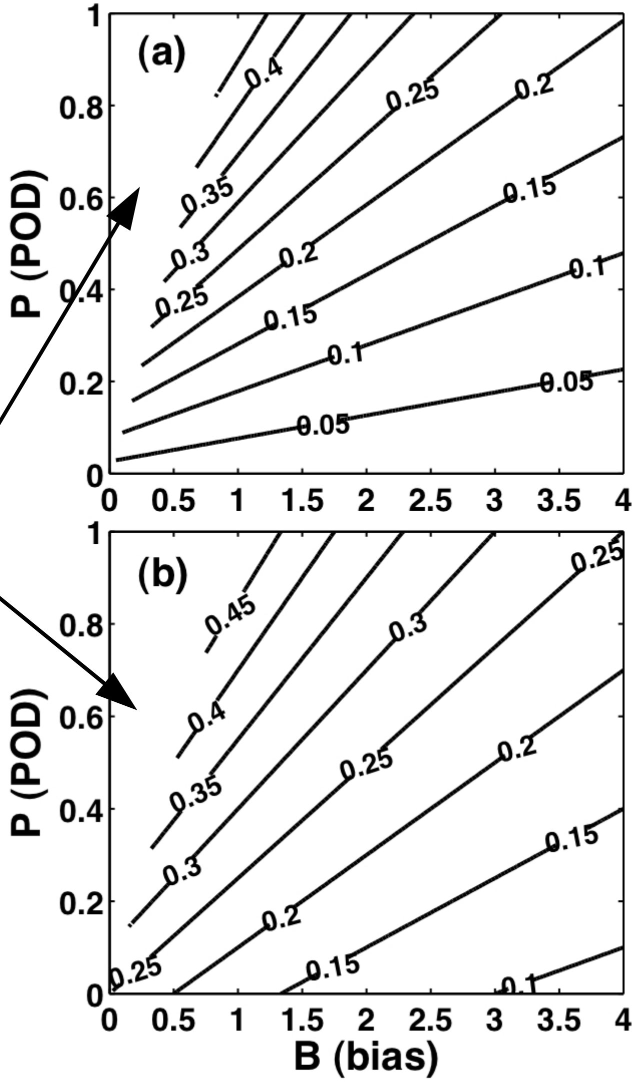
Lambert *W* function: $z = W(z)\,e^{W(z)}$. $\quad 1 = W(z)\frac{dW(z)}{dz}e^{W(z)} + \frac{dW(z)}{dz}e^{W(z)}$. $\quad \frac{dW(z)}{dz} = \frac{W(z)}{z[1+W(z)]}$.

ETS CPR Contours on the POD-Bias Plane

$\alpha = .025$

$\alpha = .25$

POD cannot exceed bias



Threat Score is similar to this one.

Table of Derivative & CPR Formulas for Selected Performance Measures

PM	$S(B,P)$	$\partial S / \partial B$	$\partial S / \partial P$	**CPR**	**CPR for** $B=1$
TS	$\frac{P}{(B+1-P)}$	$\frac{-P}{(B+1-P)^2}$	$\frac{B+1}{(B+1-P)^2}$	$\frac{P}{(B+1)}$	$\frac{P}{2}$
ETS	$\frac{P-\alpha B}{(B+1-P-\alpha B)}$	$\frac{P(2\alpha-1)-\alpha}{(B+1-P-\alpha B)^2}$	$\frac{B+1-2\alpha B}{(B+1-P-\alpha B)^2}$	$\frac{P+\alpha-2\alpha P}{(B+1-2\alpha B)}$	$\frac{P+\alpha-2\alpha P}{2(1-\alpha)}$
HSS	$\frac{2(P-\alpha B)}{(B+1-2\alpha B)}$	$\frac{2P(2\alpha-1)-2\alpha}{(B+1-2\alpha B)^2}$	$\frac{2}{B+1-2\alpha B}$	$\frac{P+\alpha-2\alpha P}{(B+1-2\alpha B)}$	$\frac{P+\alpha-2\alpha P}{2(1-\alpha)}$
CSS	$\frac{P-\alpha B}{B(1-\alpha B)}$	$\frac{2\alpha PB-P-\alpha^2 B^2}{B^2(1-\alpha B)^2}$	$\frac{1}{B(1-\alpha B)}$	$\frac{P+\alpha^2 B^2-2\alpha PB}{B(1-\alpha B)}$	$\frac{P+\alpha^2-2\alpha P}{(1-\alpha)}$
ORSS	$\frac{P-\alpha B}{D}$, $D=P-2\alpha PB-2\alpha P + 2\alpha P^2+\alpha B$	$\frac{2\alpha P(P-1)(1-\alpha)}{D^2}$	$\frac{2\alpha Y}{D^2}$, $Y=B-P^2-\alpha B^2-\alpha B+2\alpha BP$	$\frac{P(1-P)(1-\alpha)}{Y}$	$\frac{P(1-\alpha)}{(1+P-2\alpha)}$
TSA dHdF	$\frac{1-(1-P)^{1/B}}{1+(1-P)^{1/B}}$	$\frac{2(1-P)^{1/B}\ln(1-P)}{B^2[1+(1-P)^{1/B}]^2}$	$\frac{2(1-P)^{\frac{1}{B}-1}}{B[1+(1-P)^{1/B}]^2}$	$\frac{(P-1)\ln(1-P)}{B}$	$(P-1)\ln(1-P)$
ETSA dHdF	$\frac{1-\alpha-(1-P)^{1/B}}{1-\alpha+(1-P)^{1/B}}$	$\frac{2(1-\alpha)(1-P)^{1/B}\ln(1-P)}{B^2[1+(1-P)^{1/B}]^2}$	$\frac{2(1-\alpha)(1-P)^{\frac{1}{B}-1}}{B[1+(1-P)^{1/B}]^2}$	$\frac{(P-1)\ln(1-P)}{B}$	$(P-1)\ln(1-P)$
TSA dHdA	$\frac{1-u^{-1}W(u)}{1+u^{-1}W(u)}$, $u=\frac{-\ln(1-P)}{(B-P)}$	$\frac{2[W(u)]^2[1+W(u)]^{-1}}{\ln(1-P)[1+u^{-1}W(u)]^2}$	$-\frac{\partial S}{\partial B}(\frac{(1-P)u+1}{(1-P)u})$	$\frac{(P-1)\ln(1-P)}{B-P+(P-1)\ln(1-P)}$	$\frac{\ln(1-P)}{\ln(1-P)-1}$
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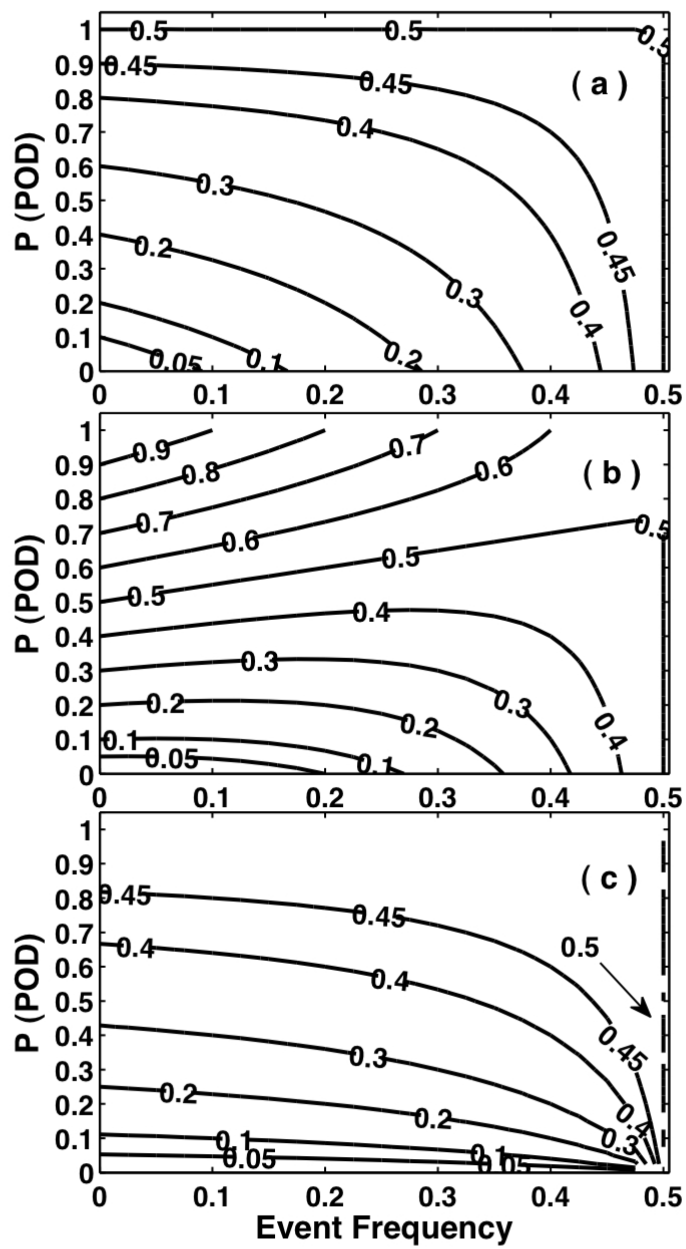
Lambert *W* function: $z = W(z)\,e^{W(z)}$. $1 = W(z)\frac{dW(z)}{dz}e^{W(z)} + \frac{dW(z)}{dz}e^{W(z)}$. $\frac{dW(z)}{dz} = \frac{W(z)}{z[1+W(z)]}$.

CPR Contours on the POD-α Plane for B=*1*

Equitable Threat Score

Clayton Skill Score

Odds Ratio Skill Score



Mesinger (2008) Bias Adjusted TS & ETS

- Uses *F, H, O* values to interpolate/extrapolate *H* to the condition *B=1,* with H_a hits.
- Presents two methods for computing H_a:
  - d*H*d*F* assumes hit area change with respect to forecast area is proportional to *(O-H).*
  - d*H*d*A* assumes hit area change with respect to false alarmed area is proportional to *(O-H).*
- Computes TS or ETS using H_a and *F=O (B=1)* to account for errors in placement.

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ETS	$\frac{P-\alpha B}{(B+1-P-\alpha B)}$	$\frac{P(2\alpha-1)-\alpha}{(B+1-P-\alpha B)^2}$	$\frac{B+1-2\alpha B}{(B+1-P-\alpha B)^2}$	$\frac{P+\alpha-2\alpha P}{(B+1-2\alpha B)}$	$\frac{P+\alpha-2\alpha P}{2(1-\alpha)}$
HSS	$\frac{2(P-\alpha B)}{(B+1-2\alpha B)}$	$\frac{2P(2\alpha-1)-2\alpha}{(B+1-2\alpha B)^2}$	$\frac{2}{B+1-2\alpha B}$	$\frac{P+\alpha-2\alpha P}{(B+1-2\alpha B)}$	$\frac{P+\alpha-2\alpha P}{2(1-\alpha)}$
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ETSA dHdF	$\frac{1-\alpha-(1-P)^{1/B}}{1-\alpha+(1-P)^{1/B}}$	$\frac{2(1-\alpha)(1-P)^{1/B}\ln(1-P)}{B^2[1+(1-P)^{1/B}]^2}$	$\frac{2(1-\alpha)(1-P)^{\frac{1}{B}-1}}{B[1+(1-P)^{1/B}]^2}$	$\frac{(P-1)\ln(1-P)}{B}$	$(P-1)\ln(1-P)$
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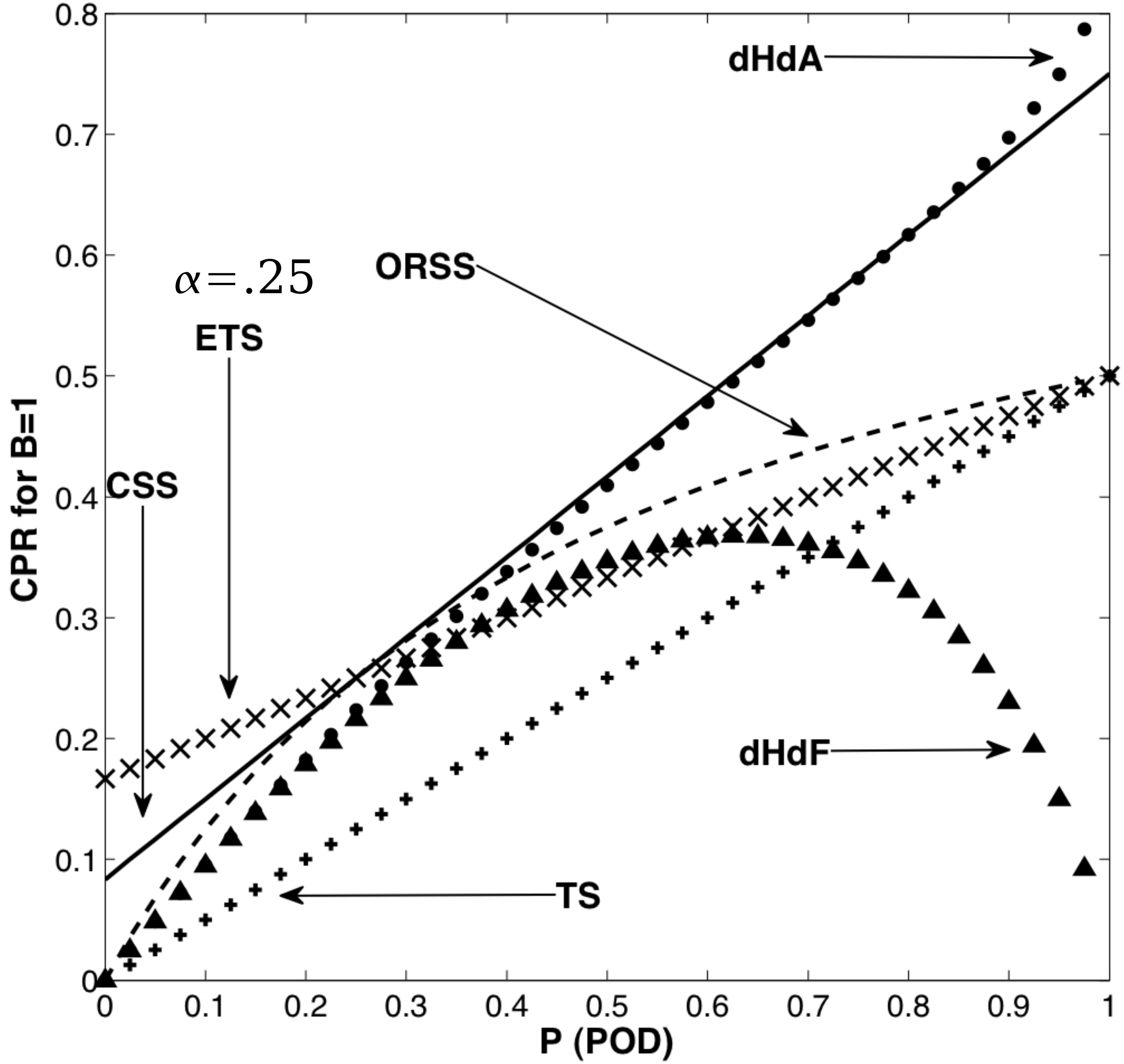
Lambert *W* function: $z=W(z)\,e^{W(z)}$. $1=W(z)\frac{dW(z)}{dz}e^{W(z)}+\frac{dW(z)}{dz}e^{W(z)}$. $\frac{dW(z)}{dz}=\frac{W(z)}{z[1+W(z)]}$.

CPR vs POD for B=1

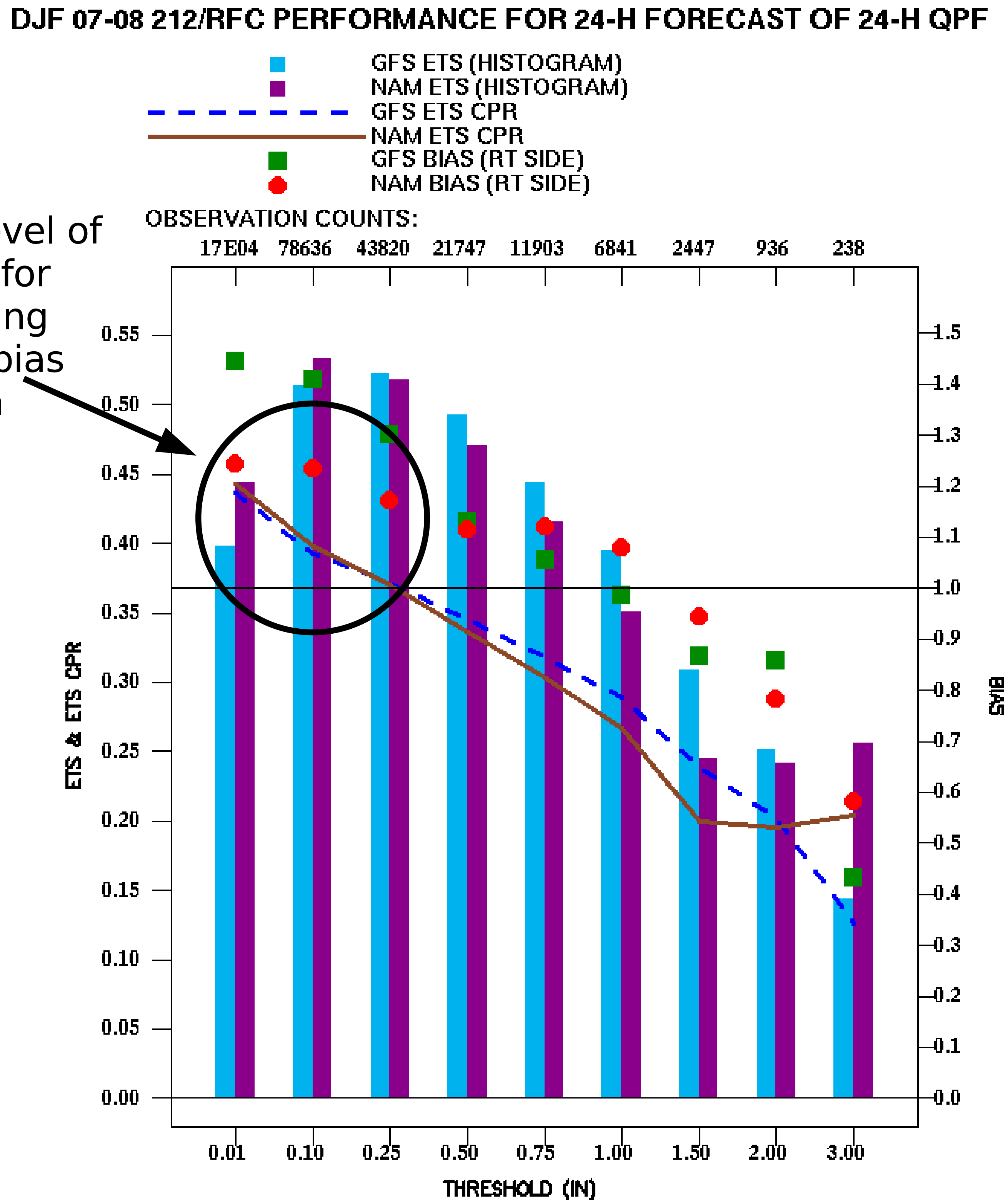


Applications

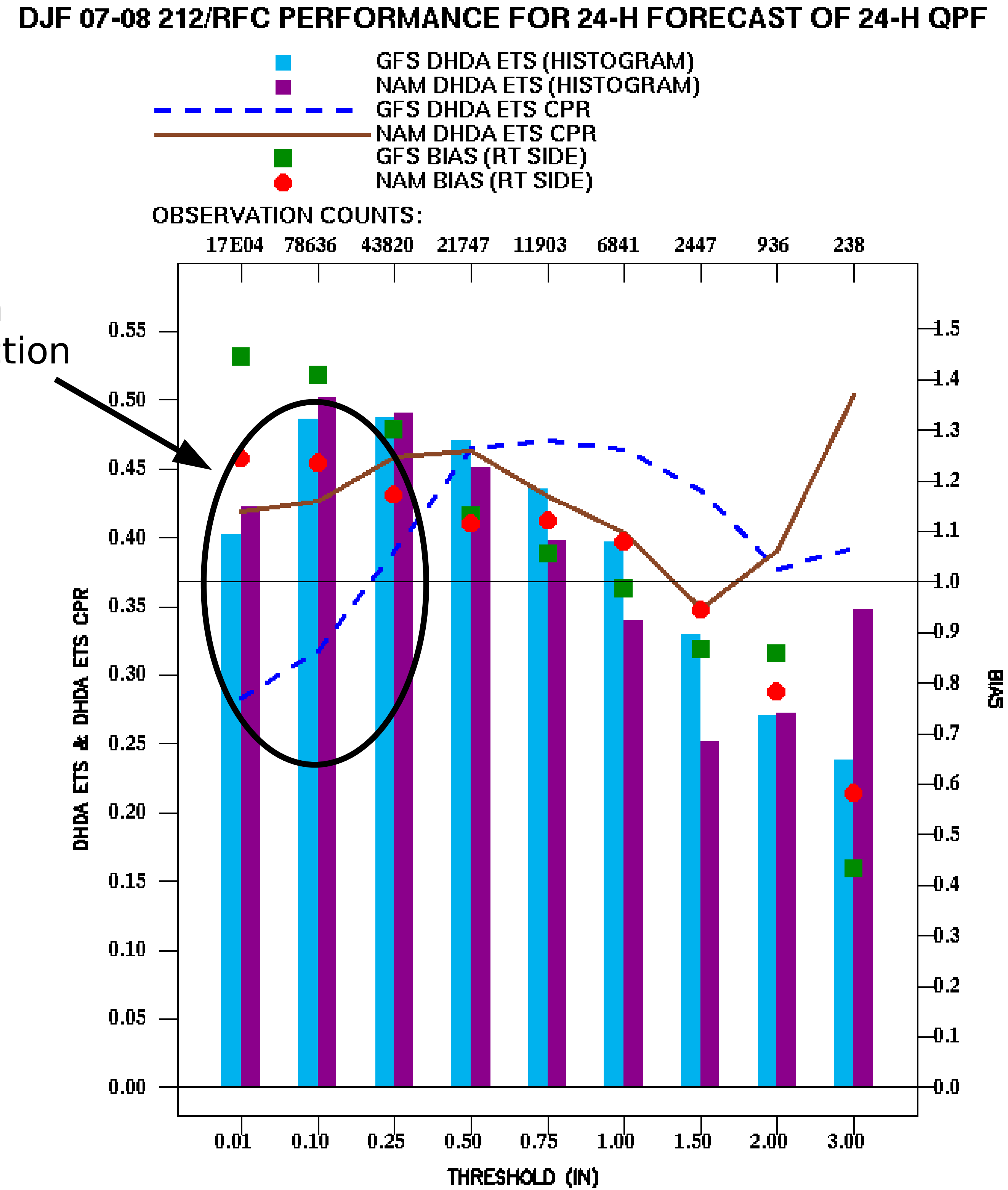
- ETS and DHDA ETS CPRs vs threshold for cold season NAM and GFS QPF.
- ETS and DHDA ETS CPRs vs threshold for warm season NAM and GFS QPF.

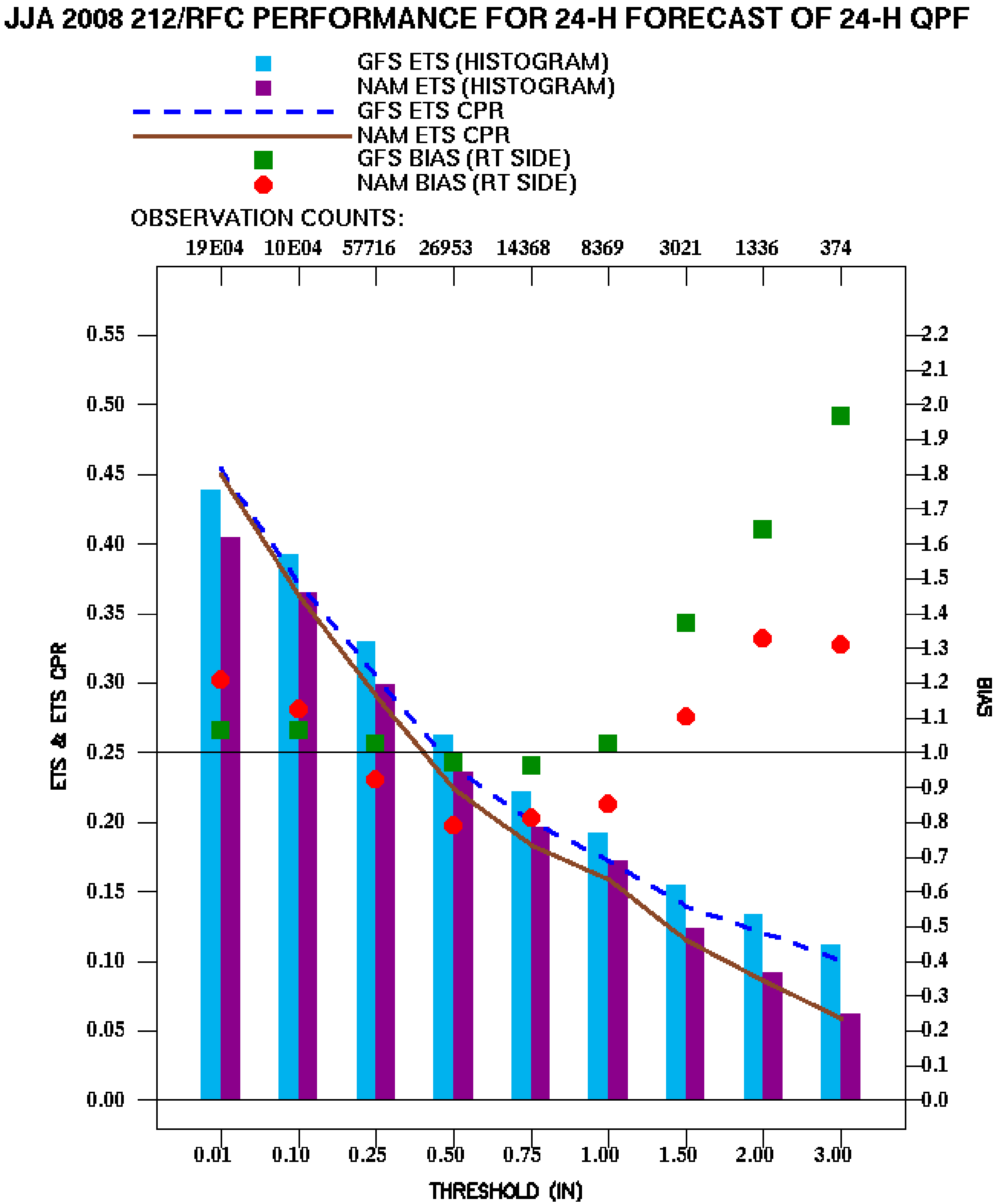
Note to fvs users: Version 2008.07 of fvs computes and displays CPR functions for all performance measures for FHO stats. Enter “fvs v” for more information on computational codes 1036--1067.

Similar level of difficulty for maintaining ETS in a bias reduction



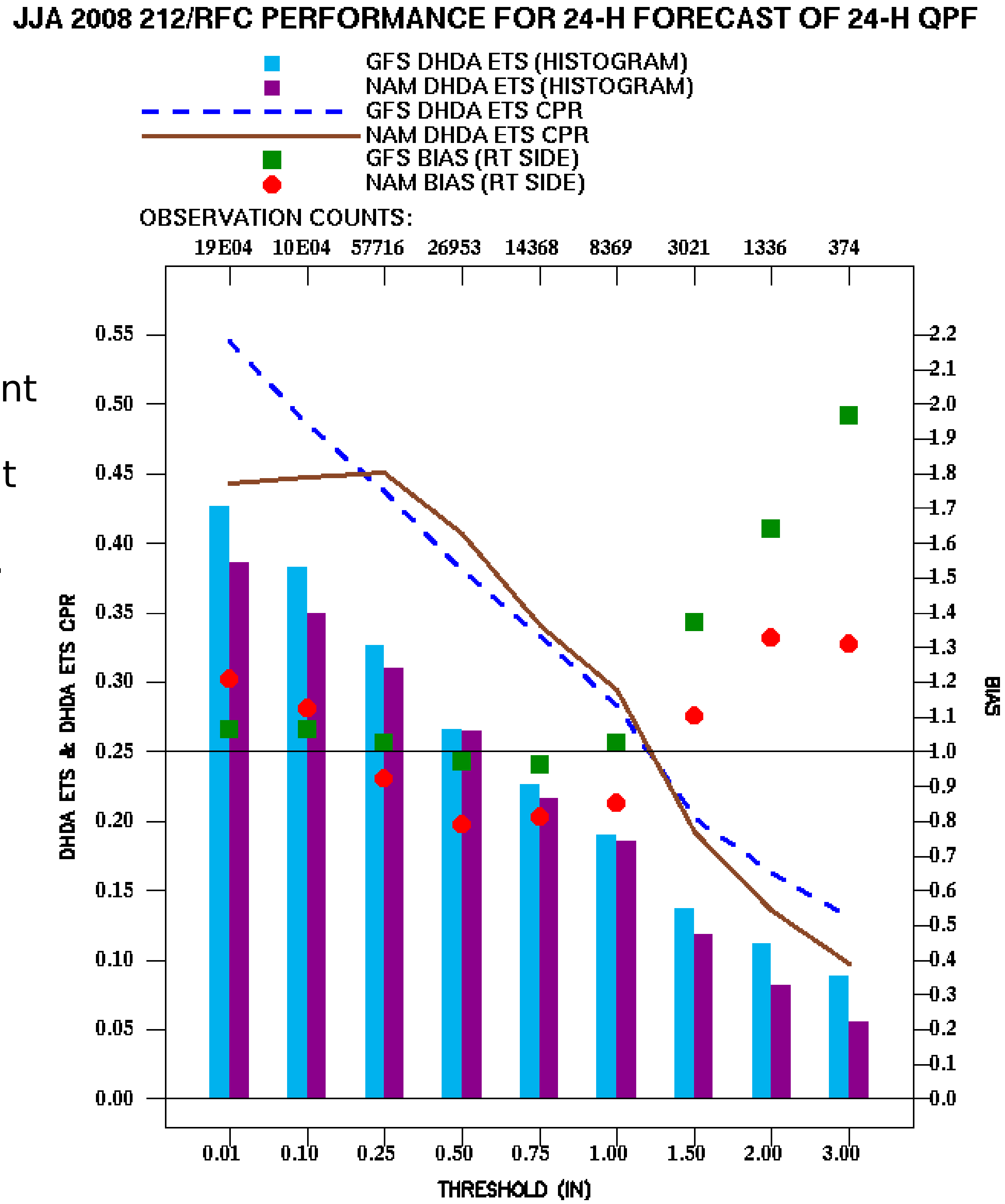
Easier to maintain DHDA ETS for NAM in bias reduction





JJA 2008 212/RFC PERFORMANCE FOR 24-H FORECAST OF 24-H QPF
GFS ETS (HISTOGRAM)
NAM ETS (HISTOGRAM)
GFS ETS CPR
NAM ETS CPR
GFS BIAS (RT SIDE)
NAM BIAS (RT SIDE)
OBSERVATION COUNTS:
19E04 10E04 57716 26953 14368 8369 3021 1336 374
ETS & ETS CPR
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55
BIAS
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2
0.01 0.10 0.25 0.50 0.75 1.00 1.50 2.00 3.00
THRESHOLD (IN)

DHDA ETS is more resistant to showing improvement if hedging inflates bias.



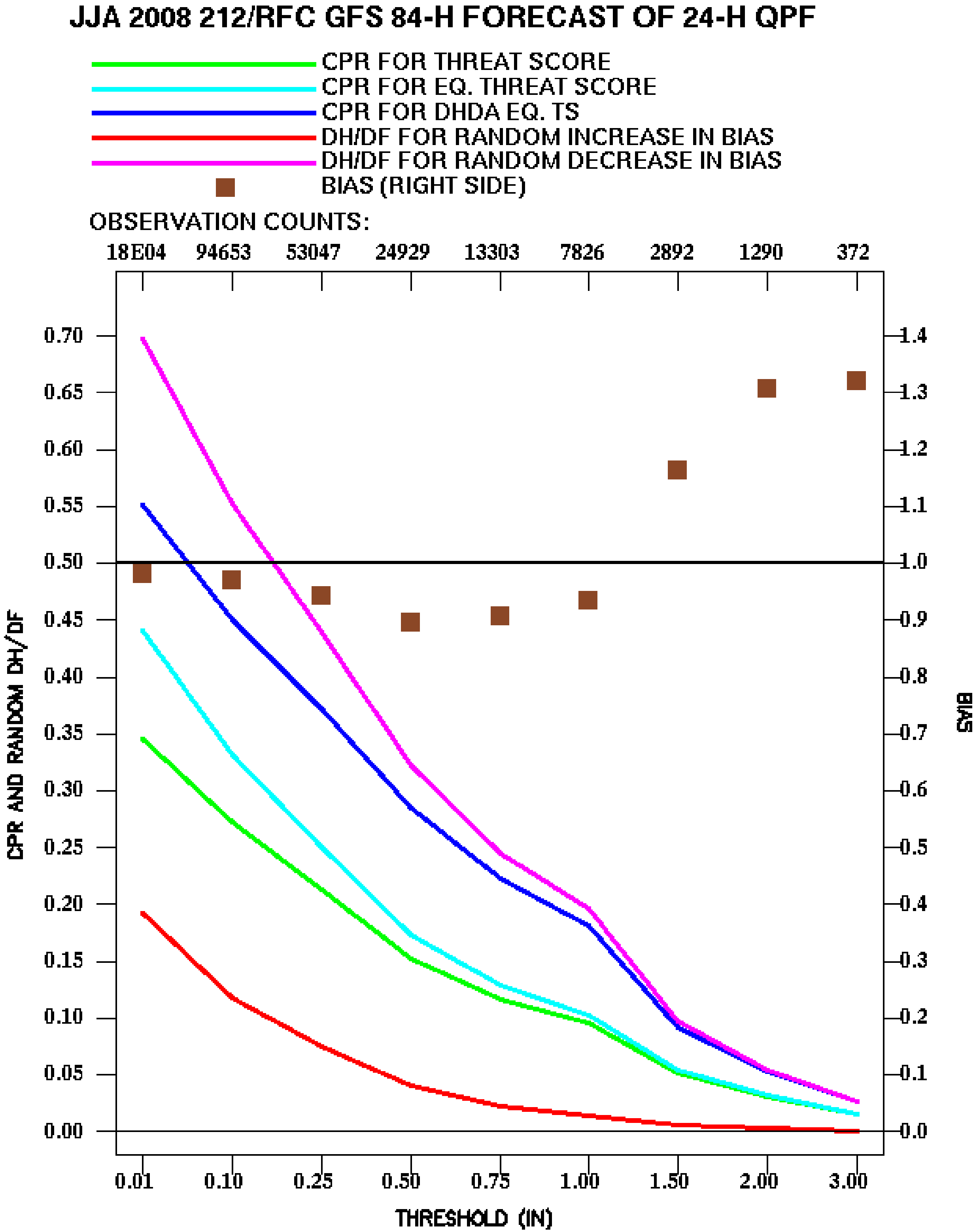
Summary

- The critical performance ratio CPR quantifies bias sensitivity.
- The CPR and, therefore, bias sensitivity depend on one or more of POD, bias, and event frequency.
- <u>All</u> performance measures are sensitive to bias.
- CPR values for Mesinger's bias adjusted ETS may give a better indication of how easily ETS can be maintained or improved in a bias correction, especially if bias is large.
- DHDA ETS is more resistant to showing improvement if hedging inflates bias.

Future Work

- Publish a note on the CPR for bias adjusted TS and ETS.
- Investigate two bench marks for the CPR
  - Two conditional probabilities have CPR expressions identical to themselves:
    - Detection Failure Ratio = the chance of randomly making hits for an increase in forecast area
    - Frequency of Hits (post agreement) = the chance of randomly loosing hits for a decrease in forecast area

T H E


JJA 2008 212/RFC GFS 84-H FORECAST OF 24-H QPF
CPR FOR THREAT SCORE
CPR FOR EQ. THREAT SCORE
CPR FOR DHDA EQ. TS
DH/DF FOR RANDOM INCREASE IN BIAS
DH/DF FOR RANDOM DECREASE IN BIAS
BIAS (RIGHT SIDE)
OBSERVATION COUNTS:
18E04 94653 53047 24929 13303 7826 2892 1290 372
0.70 0.65 0.60 0.55 0.50 0.45 0.40 0.35 0.30 0.25 0.20 0.15 0.10 0.05 0.00
CPR AND RANDOM DH/DF
1.4 1.3 1.2 1.1 1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
BIAS
0.01 0.10 0.25 0.50 0.75 1.00 1.50 2.00 3.00
THRESHOLD (IN)

E N D